a) #1 $36 + 12 + 4 = 52$ miles

$C = 5(52)$

$= \boxed{26,000,000}$

- Add up the distance to reach the refinery
- Multiply distance by the cost per mile to get the total.

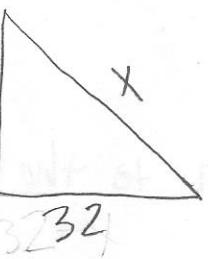It would cost $26,000,000 to reach the refinery going west.

#2 $32 + 12 = 44$

$20 + 3.2 + 4(1.2) = 28$

$C = 5(44) + 28 = \boxed{24,800,000}$

- Add the distance to reach the refinery
- Add up all the costs to go through the mountain.
- Multiple distance and cost per mile, then add the cost to go through the mountain for the total cost.

It would cost $24,800,000 to go through the mountain to reach the refinery.

b) #1

$12^2 + 32^2 = X$

$X = \sqrt{1168}$

≈ 34.176 mi

$C = 8.5(\sqrt{1168})$

$= \boxed{29,049,613}$

- Find the distance that is traveled through private ground.
- Multiply the distance by the cost to go through private land per mile to get the total cost.

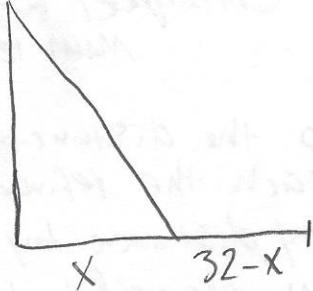It would cost $29,049,613 to directly from the well to the refinery.

#2 $C = 8.5(12) + 5(32)$

$= \boxed{26,200,000}$

- Multiply the distance through private by the cost to go through per mile and add that to the distance outside private land multiplied by the cost per mile for the total cost.

It would cost $26,200,000 to go directly south then east once out of the private land and to the refinery.

c)

$12^2 + x^2 = D^2$

$D = \sqrt{144 + x^2}$

$$C(x) = 8.5\left(\sqrt{144 + x^2}\right) + 5(32 - x)$$

- Use the pythagorean theorem to find the distance of in the private land.
- Create the cost function in terms of x.

d) $C'(x) = \dfrac{8.5}{2\sqrt{144 + x^2}}(2x) - 5$

$\quad = \dfrac{8.5x}{\sqrt{144 + x^2}} - 5 = 0$

- Find the derivative of the cost function and set equal to zero
- Solve for x

$\dfrac{8.5x}{\sqrt{144 + x^2}} = 5$

$\left(8.5x = 5\sqrt{144 + x^2}\right)^2$

$72.25x^2 = 25(144 + x^2)$

$72.25x^2 = 3600 + 25x^2$

$47.25x^2 = 3600$

$\sqrt{x^2} = \sqrt{\dfrac{3600}{47.25}}$

$x \approx 8.7287 \text{ miles}$

$C(8.7287) = 8.5\left(\sqrt{144 + 8.7287^2}\right) + 5(32 - 8.7287)$

$\quad = 24,248,636$

- Plug x in to the cost function

It would cost $24,248,636 to cut across the private ground for approximately 8.7 miles and then head directly east along the border of the private land.